

Waste thermal energy harvesting from a convection-driven Rijke–Zhao thermo-acoustic-piezo system

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ABSTRACT

In this work, a convection-driven Rijke–Zhao thermo-acoustic-piezo system is designed and experimentally tested to demonstrate its potential for harvesting thermal energy. For this, a nonlinear theoretical model is developed to simulate the energy conversion process, i.e. heat-to-sound-to-electricity in the present system. Unlike the conventional conduction-driven thermoacoustic converters, our present system involves no heat exchangers and stacks. As a heat source is placed in a Rijke–Zhao tube with two bifurcating daughter branches, self-sustained thermoacoustic oscillations are generated. The resulting acoustic fields in the bifurcating branches are dramatically different. One branch is associated with ‘hot’ oscillations. However the other is with ‘cold’ oscillations at ambient temperature, which enable a piezo-electric generator being implemented to the end of the branch. In order to measure the acoustic fields in the bifurcating branches, two arrays of thermocouples and microphones are used. The maximum sound pressure level is around 139 dB. The output electric power and acoustical energy conversion efficiency are measured and compared with that from a similar but a conduction-driven thermo-acoustic-piezo system. It is found that 60% more power is generated. And the energy conversion efficiency is increased by 105%. These experimental results confirm that the developed Rijke–Zhao thermo-acoustic-piezo system is an invaluable tool in designing a simple, low-cost, energy-efficient thermoacoustic system.

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1. Introduction

Greenhouse gas emission and fossil fuel exhaustion are increasingly urgent issues. However, industry waste heat generated by burning fossil fuel is released to the atmosphere. This is not only harmful to the environment, but also energy wasting. To recover waste heat, there is a need for energy conversion techniques to convert heat into electricity or mechanical work, especially for heat at low temperature. Thermo-acoustic energy conversion technology [1,2] may be used for energy harvesting. Generally, thermoacoustic energy conversion systems are associated with two distinct types of oscillations: (i) convection-driven, or (ii) conduction-driven. Rijke tube [1–6] is a typical example of convection-driven thermoacoustic systems. It is a simple open ended vertical tube with a confined heat source in its lower half. And a net flow of gas through the tube is generated due to the natural convection, which is essential in producing self-sustained oscillations. Thus the Rijke oscillations are convection-driven. Sondhauss tube [4,5] is a classical device of producing conduction-driven oscillations. It is a pipe having one closed and one open end. When heat is added and transferred through the wall of the closed end via conduction,

Sondhauss oscillations occur. There is no net flow of gas through the tube and so the Sondhauss oscillations are conduction-driven in the presence of mean temperature gradients [7].

To elucidate thermoacoustic energy conversion process [8,9], as occurred in Rijke- and Sondhauss-tube as shown in Fig. 1, considerable research has been carried out. The first insight into thermoacoustic oscillations was provided by Rayleigh [10] over a century ago. He stated that intensive thermoacoustic oscillations would occur when heat was added in phase with pressure, but would not occur if heat was added out of phase with pressure. Culick [11] derived a general form of the Rayleigh's criterion, which is applicable to both linear and nonlinear thermoacoustic oscillations in any shaped chamber. He showed that an oscillating heat source functioned like an oscillating piston. The same conclusion was drawn by Chu and Ying [12] and Rott [8]. Recently, Bisio and Rubatto [6] reviewed the heat-generated acoustic oscillations in Sondhauss and Rijke tubes and discussed their possible applications.

Besides offering insight into the energy exchange that gives rise to and limits the growth of intensive oscillations, Rayleigh criterion also suggests ways in which thermoacoustic oscillations can be maximized or minimized, depending on applications in practice. One application in aero-engines and gas turbines is to minimize convection-driven thermoacoustic oscillations to stabilize combustion

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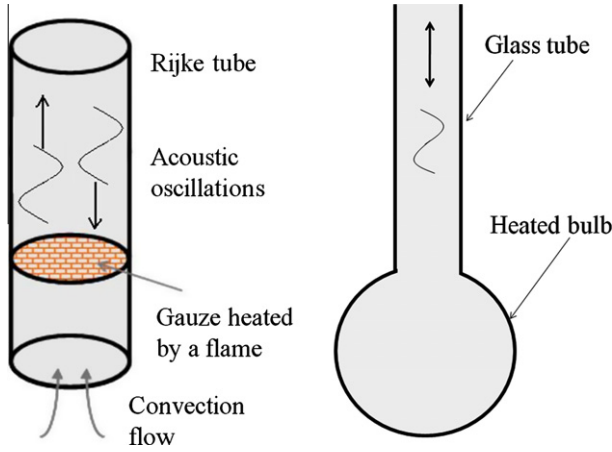


Fig. 1. A typical Rijke tube with a convection flow and Sondhauss tube.

diaphragm to acoustic oscillations. It is worth noting that the present system is different from conventional Rijke tube due to the unique structure and geometric configuration and the resulting flow patterns in the bifurcating branches [39]. It will be shown later that the present system can produce both ‘hot’ and ‘cold’ thermoacoustic oscillations, which enables the energy harvesting achievable by using piezoelectric generator. However, conventional Rijke tube produces only ‘hot’ oscillations.

2.1. Model for pressure wave generation by unsteady heat release

The heat source in our experiment is a premixed laminar flame. The presence of the heat source separate the Rijke–Zhao combustor into 4 regions, as shown in Fig. 2. The flame is assumed to be acoustically compact and so it is modelled as a thin sheet. The flow fluctuations upstream and downstream of the flame can be modelled as being due to plane acoustic waves travelling in opposite directions. Thus the equations for the flow fluctuations upstream and downstream of the flame are expressed in terms of acoustic waves R_i and L_i as given

$$p_i(z, t) = \bar{p} + R_i \left(t - \frac{z}{\bar{c}_i + \bar{u}_i} \right) + L_i \left(t + \frac{z}{\bar{c}_i - \bar{u}_i} \right) \quad (1a)$$

$$u_i(z, t) = \bar{u}_i + \frac{1}{\rho_i \bar{c}_i} \left[R_i \left(t - \frac{z}{\bar{c}_i + \bar{u}_i} \right) - L_i \left(t + \frac{z}{\bar{c}_i - \bar{u}_i} \right) \right] \quad (1b)$$

$$\rho_i(z, t) = \bar{\rho}_i + \frac{1}{\bar{c}_i^2} \left[R_i \left(t - \frac{z}{\bar{c}_i + \bar{u}_i} \right) + L_i \left(t + \frac{z}{\bar{c}_i - \bar{u}_i} \right) \right] \quad (1c)$$

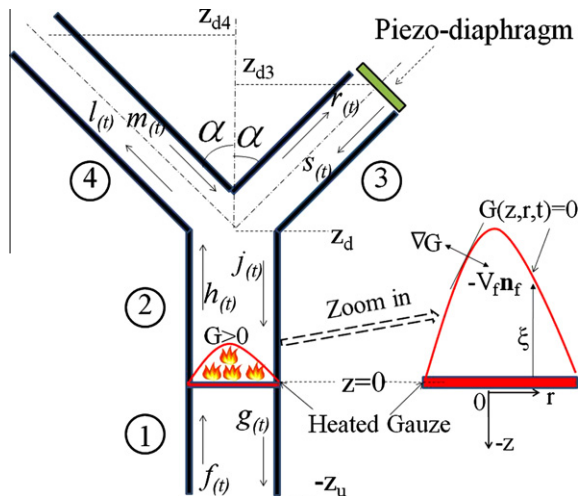


Fig. 2. Schematic of a Rijke–Zhao thermo-acoustic-piezo system.

where p denotes the instantaneous pressure, u is the flow velocity, ρ is the density, c the speed of sound and subscript i denotes the i th zone. L_i denotes the incident wave $g(t)$, $j(t)$, $m(t)$ or $s(t)$, and R_i denotes the reflected one $f(t)$, $h(t)$, $r(t)$ or $l(t)$ in the different regions. The instantaneous value consists of a mean value denoted by an overbar, and a perturbation part denoted by a prime. $R_i(t) = \bar{R}_i(\omega)e^{j\omega t}$ and $L_i(t) = \bar{L}_i(\omega)e^{j\omega t}$ denote the reflected and incident wave respectively. Here flow disturbances are assumed to be time-dependent $e^{j\omega t}$.

The momentum and energy equations across the flame hold as

$$p_2 - p_1 + \rho_2 u_2 (u_2 - u_1) + \frac{1}{2} c_D \rho_1 u_1^2 = 0 \quad (2)$$

$$\frac{\gamma}{\gamma - 1} (p_2 u_2 - p_1 u_1) + \frac{1}{2} \rho_1 u_1 (u_2^2 - u_1^2) = \frac{\bar{Q} + Q'(t)}{A_1} \quad (3)$$

The term on right hand side of Eq. (3) describes the heat release from a given heat source. To capture the heat release dynamics, a nonlinear flame model is developed. The flame model describes a premixed laminar flame, burning in the wake of a metal wire gauze anchored to a Bunsen burner in Rijke–Zhao tube. The fluctuations of the oncoming flow velocity at the gauze result in flame area variations and further lead to unsteady heat release, which is an efficient sound source. The instantaneous total rate of heat release $Q(t) = \bar{Q} + Q'(t)$ has been shown [14] to depends on the flame surface area $A_f(t) = \bar{A}_f + A'_f$ as

$$\frac{\bar{Q} + Q'(t)}{\bar{Q}} = \frac{\bar{A}_f + A'_f(t - \tau_v)}{\bar{A}_f} \quad (4)$$

where τ_v represents a time delay between $Q(t)$ and $A_f(t)$, which depends on the flame surface front.

The conical flame surface is assumed to propagate itself in the direction of its normal $\mathbf{n}_f = \nabla G / |\nabla G|$, into the unburnt fluid at a constant laminar burning velocity V_f relative to the unburnt gas. Here the flame surface axial position at radius r is given by $\mathbf{G}(z, r, t) = z - \xi(r, t) = 0$. z is the axial coordinate and t is time. As shown in Fig. 2, the surface $\mathbf{G} = 0$ moves in the direction of its normal with speed $\mathbf{u}_f - V_f \mathbf{n}_f$, where $\mathbf{u}_f = (u, v)$ is the unburnt gas velocity, and $\mathbf{n}_f$ points upstream. Mathematically this statement is equivalent to $d\mathbf{G}/dt = 0$:

$$\frac{d\mathbf{G}}{dt} = \frac{\partial \mathbf{G}}{\partial t} + (\mathbf{u}_f - V_f \mathbf{n}_f) \cdot \nabla \mathbf{G} \quad (5)$$

After substituting $\mathbf{n}_f = \nabla G / |\nabla G|$ and $\mathbf{G}(z, r, t) = z - \xi(r, t)$, the familiar G-equation is obtained, which describe the location of the flame surface [14]. Substituting for $\mathbf{G}(x, r, t) = z - \xi(r, t)$ gives

$$\frac{\partial \xi}{\partial t} + v \frac{\partial \xi}{\partial r} + V_f \sqrt{1 + \left(\frac{\partial \xi}{\partial r} \right)^2} = u \quad (6)$$

Since the fuel–air ratio is so low for the laminar flame that the overall expansion is negligible. Thus the density change across the surface $\mathbf{G}(z, r, t) = 0$ is assumed to be negligible. The particle velocity in Eq. (6) is given by that in the oncoming flow $(u_c, 0)$. Eq. (6) then simplifies to:

$$\frac{\partial \xi}{\partial t} = u_c - V_f \sqrt{1 + \left(\frac{\partial \xi}{\partial r} \right)^2} \quad (7)$$

To solve Eq. (7) so as to determine the flame front location $\xi(r, t)$ for a specified oncoming velocity, numerical method such as Runge–Kutta is needed. For simplicity the oncoming velocity is assumed to be independent of radius, i.e. $u_c(t)$ as given as

$$u_c = \bar{u}_c + \frac{1}{\rho_1 \bar{c}_1} (f(t) - g(t)) \quad (8)$$

Once $\xi(r, t)$ is known, the instantaneous flame surface area $A_f(t)$ can be readily calculated, since

$$A_f(t) = \int_0^{2\pi} d\theta \int_0^R r \sqrt{1 + \left(\frac{\partial \xi}{\partial r}\right)^2} dr \quad (9)$$

Thus linearizing the momentum and energy Eqs. (2) and (3) gives the time evolution of the flow fluctuations $f(t)$, $g(t)$, $h(t)$ and $j(t)$ generated by the unsteady heat release $Q'(t)$.

At the junction section of the bifurcating branches, mass and energy conservation hold as

$$\frac{\partial}{\partial t} \int_{\delta V} \rho dV = \rho_2 u_2 A_2 - \rho_3 u_3 A_3 + \rho_4 u_4 A_4 = 0 \quad (10a)$$

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\delta V} \left(\frac{1}{2} \rho u^2 + \rho c_p T \right) dV = & \rho_3 u_3 A_3 \left(\frac{1}{2} u_3^2 + c_p T_3 \right) \\ & + \rho_4 u_4 A_4 \left(\frac{1}{2} u_4^2 + c_p T_4 \right) - A_2 \rho_2 u_2 \left(\frac{1}{2} u_2^2 + c_p T_2 \right) = 0 \end{aligned} \quad (10b)$$

where A_1 , A_2 , A_3 and A_4 denote the cross-sectional area of the combustor in different regions. Linearizing the mass and energy Eqs. (10a) and (10b) and expressing the flow fluctuations in terms of $r(t)$, $s(t)$, $l(t)$ and $m(t)$ gives the time evolution of the acoustic waves propagated in bifurcating upper branches. However, for convenience, pressure continuity can be assumed to hold at the junction section, i.e.

$$\hat{h}(\omega) e^{-j\omega \frac{z_d}{c_2(1+\bar{M}_2)}} + \hat{j}(\omega) e^{j\omega \frac{z_d}{c_2(1-\bar{M}_2)}} = \hat{r}(\omega) + \hat{s}(\omega) = \hat{l}(\omega) + \hat{m}(\omega) \quad (11)$$

where overhat denotes Fourier transform of the corresponding parameter in time domain.

The combustor boundaries at the ends of the mother tube and the bifurcating branches are modelled using pressure reflection coefficients $\mathcal{Z}_u$, $\mathcal{Z}_3$ and $\mathcal{Z}_4$. In frequency domain, they correspond to $\hat{\mathcal{Z}}_u(\omega)$, $\hat{\mathcal{Z}}_3(\omega)$ and $\hat{\mathcal{Z}}_4(\omega)$ respectively [14]. These reflection coefficients relate the reflected wave to the incident wave at the boundary, as

$$\begin{aligned} \frac{\hat{f}(\omega)}{\hat{g}(\omega)} &= -\hat{\mathcal{Z}}_u(\omega) e^{-j\omega \tau_g}, \quad \frac{\hat{s}(\omega)}{\hat{r}(\omega)} = -\hat{\mathcal{Z}}_3(\omega) e^{-j\omega \tau_r}, \\ \frac{\hat{m}(\omega)}{\hat{l}(\omega)} &= -\hat{\mathcal{Z}}_4(\omega) e^{-j\omega \tau_l} \end{aligned} \quad (12)$$

For convenience, the value -1.0 can be used at the open upstream and downstream end, i.e. $\mathcal{Z}_u = \mathcal{Z}_4 = -1.0$. To make our analysis more general, the pressure reflection coefficient at the end with piezoelectric diaphragm attached is assumed to be $\mathcal{Z}_3$.

Thus the end boundary conditions and the flow conservation equations across the flame and the junction section of the bifurcating branches provide enough information to solve for each of the eight wave strengths in Fig. 2. The resulting matrix equation is given as

$$\mathbf{U} \begin{pmatrix} h(t) \\ g(t) \\ j(t) \\ r(t) \\ l(t) \end{pmatrix} = \mathbf{V} \begin{pmatrix} h(t - \tau_h) \\ g(t - \tau_g) \\ j(t - \tau_j) \\ r(t - \tau_r) \\ l(t - \tau_l) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ Q'(t)/A_1 \bar{c}_1 \end{pmatrix} \quad (13)$$

where $\mathbf{U}$ and $\mathbf{V}$ are coefficient matrices as given in Appendix. The time delays are given as $\tau_h = \frac{z_d}{c_2(1+\bar{M}_2)}$, $\tau_g = \frac{2z_u}{c_1(1-\bar{M}_1^2)}$, $\tau_j = \frac{z_d}{c_2(1-\bar{M}_2)}$, $\tau_r = \frac{2(z_{d4}-z_d)/\cos \alpha}{c_3(1-\bar{M}_3^2)}$, and $\tau_l = \frac{2(z_{d4}-z_d)/\cos \alpha}{c_4(1-\bar{M}_4^2)}$, $\bar{M}_i$ the mean Mach number in i th region of the Rijke–Zhao combustor.

Now the time evolution of the flow perturbations from specified initial conditions can be determined by numerically solving

Eqs. (7), (9) and (13) using a fourth-order Runge–Kutta method. In our calculations, $\xi(r, t)$ was evaluated at 50 locations across the duct radius $R = 0.025$ m, the length of the mother tube is 0.3 m, the bifurcating branches are 0.5 and 0.9 m respectively and the flame is placed at 0.13 m away from the bottom open end.

Fig. 3a shows that the time evolution of the heat-driven acoustic oscillation. It can be seen that initial flow disturbance is quickly grown into limit cycle. The resulting thermoacoustic oscillation is at around 170 Hz with a sound pressure level 138 dB. Harmonics are also observed, as shown in Fig. 3b. Fig. 3c shows the insightful information about the nonlinear periodic oscillation in the phase-space diagram between pressure and velocity. Fig. 3d illustrates the variation of the phase difference $\theta(\omega_1, t)$ between the pressure $p'(t)$ and heat release $Q'(t)$ at frequency ω with time, which is defined as

$$\theta(\omega, t) = \arccos \left(\frac{\int_0^t \frac{p'(\omega, \Theta) Q'(\omega, \Theta) d\Theta}{\sqrt{\int_0^{2\pi/\omega} p'^2(\omega, \Theta) d\Theta} \sqrt{\int_0^{2\pi/\omega} Q'^2(\omega, \Theta) d\Theta}} d\Theta \right) \quad (14)$$

It can be seen that when initial pressure perturbation grows into a limit cycle, unsteady heat release is in phase with the pressure fluctuation, i.e. $|\theta(\omega_1, t)| \approx 65^\circ < 90^\circ$. This phase information is consistent with Rayleigh's criterion [18], verifies our numerical model and reveals the generation mechanism of heat-driven acoustic oscillations.

The time evolution of the heat-driven velocity oscillation is shown in Fig. 9. By comparing the estimation with the measurement, good agreement is observed. This validate our present heat-driven acoustic wave model.

2.2. Model for electricity generation by deformed piezoelectric diaphragm

Piezoelectric materials produce a voltage in response to an applied force because of charge redistribution caused by deformation. Similarly, a change in dimensions can be induced by the application of a voltage to a piezoelectric material. In our application, the acoustic pressure oscillations is applied to a circular piezoelectric diaphragm, which is deformed and leads to voltage output. Following the work [23,40], the coupling between the acoustic and electrical fields from the a given point on the piezo-diaphragm is formulated into a two-port model in frequency domain as

$$\begin{pmatrix} \hat{z}(\omega) \\ \hat{q}(\omega) \end{pmatrix} = \underbrace{\begin{pmatrix} \hat{H}_{11}(\omega) & \hat{H}_{12}(\omega) \\ \hat{H}_{21}(\omega) & \hat{H}_{22}(\omega) \end{pmatrix}}_{\mathbf{H}(\omega)} \begin{pmatrix} \Delta \hat{p}_p(\omega) \\ \hat{V}(\omega) \end{pmatrix} \quad (15)$$

where $\hat{z}(\omega)$, $\hat{q}(\omega)$, $\Delta \hat{p}_p(\omega)$ and $\hat{V}(\omega)$ denotes the piezo-diaphragm frequency-dependent center point deflection, charge, pressure difference and voltage. The transfer function matrix $\mathbf{H}(\omega)$ consists of the elements of the admittance of the piezo-diaphragm. Here $\hat{H}_{11}(\omega)$ and $\hat{H}_{22}(\omega)$ are the acoustical and the electrical admittances, and $\hat{H}_{21}(\omega)$ and $\hat{H}_{12}(\omega)$ denote the coupled electro-acoustic admittances. Detailed information about how to estimate the transfer function $\hat{H}(\omega)$ can be found in the work of [23]. Manipulating Eq. (15) gives the transfer function between voltage output and pressure fluctuation as

$$\frac{\hat{V}(\omega)}{\Delta \hat{p}_p(\omega)} = \frac{\hat{H}_{21}(\omega) - \hat{H}_{11}(\omega)}{\hat{H}_{12}(\omega) - \hat{H}_{22}(\omega)} + \frac{1}{\hat{H}_{12}(\omega) - \hat{H}_{22}(\omega)} \left[\frac{\hat{z}(\omega)}{\Delta \hat{p}_p(\omega)} - \frac{\hat{q}(\omega)}{\Delta \hat{p}_p(\omega)} \right] \quad (16)$$

The displacement of the thin circular piezoelectric diaphragm $z(r, \theta, t) = \hat{z}(r, \theta) \exp(j\omega t)$ results from the pressure difference between the two sides of the diaphragm, i.e. $\Delta p_p(r, \theta, t) = \Delta \hat{p}_p(r) \exp(j\omega t)$, as shown,

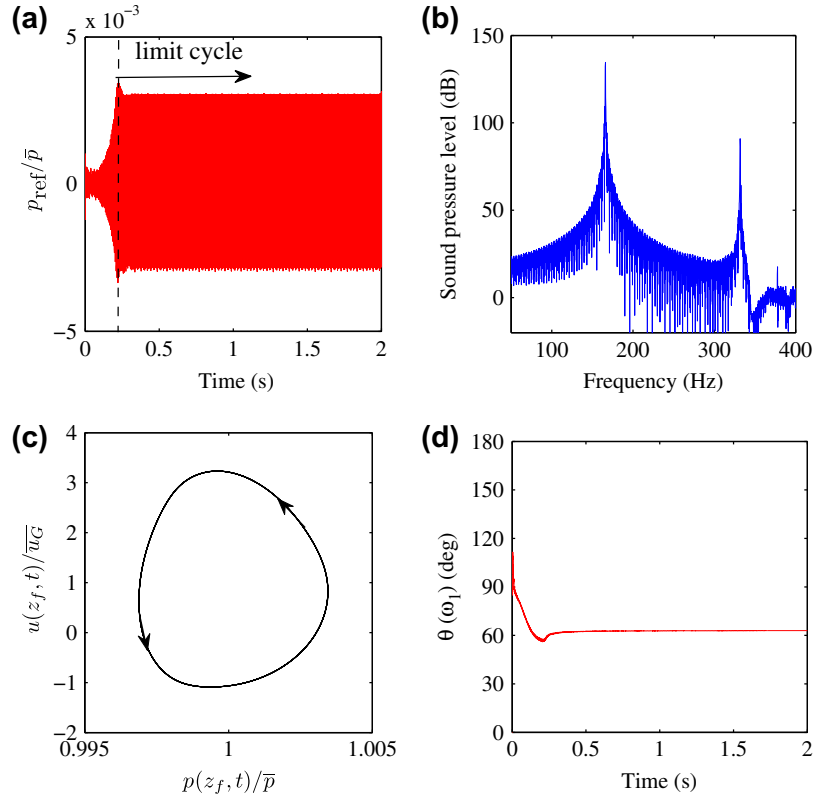


Fig. 3. Onset of Rijke–Zhao thermoacoustic oscillation: (a) Time evolution of pressure oscillation, (b) frequency spectra, (c) phase diagram between $u(z_f, t)/\bar{u}_G$ and $p(z_f, t)/\bar{p}$ and (d) phase diagram between $Q(t)/\bar{Q}$ and $p(z_f, t)/\bar{p}$.

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\zeta_m}{m_m} \frac{\partial}{\partial t} + \frac{k_m}{m_m} \right) \hat{z}(r, \theta) e^{i\omega t} - \frac{\Gamma_m}{\bar{\rho}_m} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} (\hat{z}(r, \theta)) \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} (\hat{z}(r, \theta)) \right] e^{i\omega t} = \frac{\Delta \hat{p}_p}{\bar{\rho}_m} e^{i\omega t} \quad (17)$$

where m_m , k_m , ζ_m , Γ_m and $\bar{\rho}_m$ are the mass, stiffness, damping, tension and surface density of the piezoelectric diaphragm.

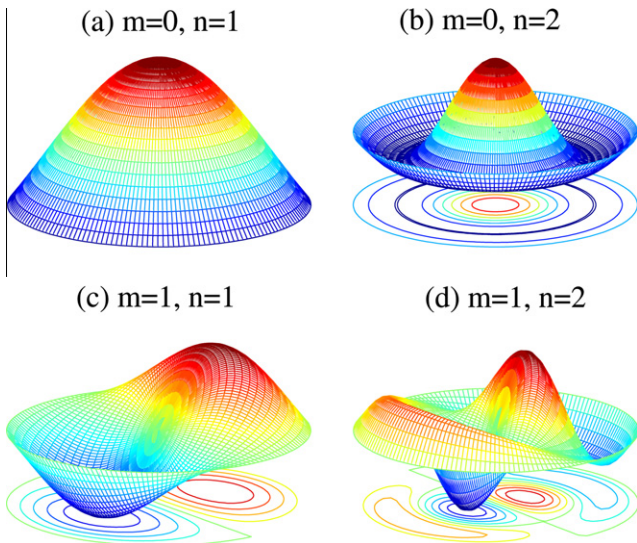


Fig. 4. The green's function for a circular piezoelectric diaphragm fixed at the rim $r = a$, when $\alpha = 1.0$, $a = 1.0$ and $\rho = 0.3$.

Eq. (17) is an inhomogeneous equation describing the diaphragm motion, which are subjected to the boundary conditions $z(a, \theta, t) = 0$. Eliminating the time dependence $e^{i\omega t}$ from both sides of Eq. (17) leads to

$$\frac{1}{r} \frac{\partial \hat{z}(r, \theta)}{\partial r} + \frac{\partial^2 \hat{z}(r, \theta)}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \hat{z}(r, \theta)}{\partial \theta^2} + \kappa^2 \hat{z}(r, \theta) = \frac{\Delta \hat{p}_p}{\bar{\rho}_m c_m^2} \quad (18)$$

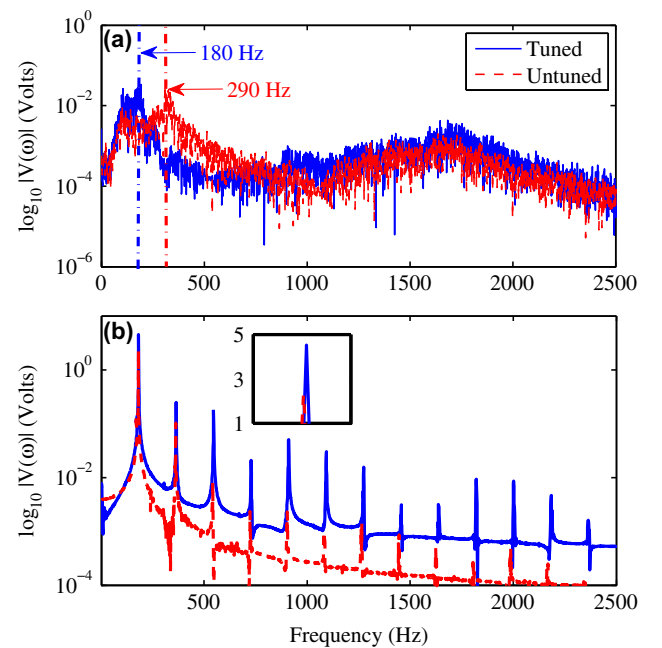


Fig. 5. Measured dynamic response of the piezoelectric diaphragm before and after tuning: (a) resonant frequency shift and (b) spectrum of the generated voltage.

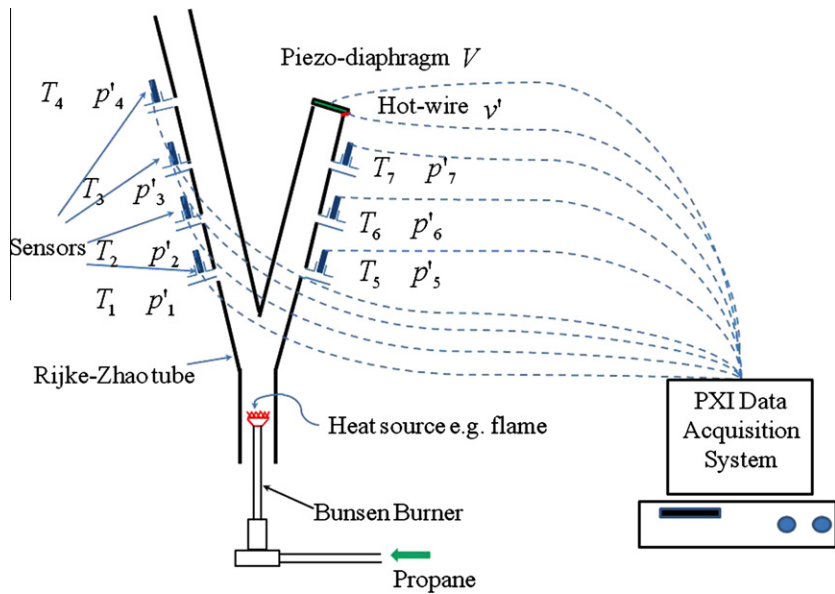


Fig. 6. Schematic of Rijke–Zhao thermoacoustic tube with a piezoelectric generator attached.

Table 1
Piezoelectric transducer parameters and dimensions of the Rijke–Zhao tube.

Weight (grams)	Stiffness (N/m)	Capacitance (nF)	Resonant frequency (Hz)	Diameter (mm)
9.8	5×10^3	107	290	63.5
α (degree)	Length z_u/z_d	Length z_{d3}/z_d	Length z_{d4}/z_d	Diameter A (m ²)
15	0.5	2.41	4.34	0.0079

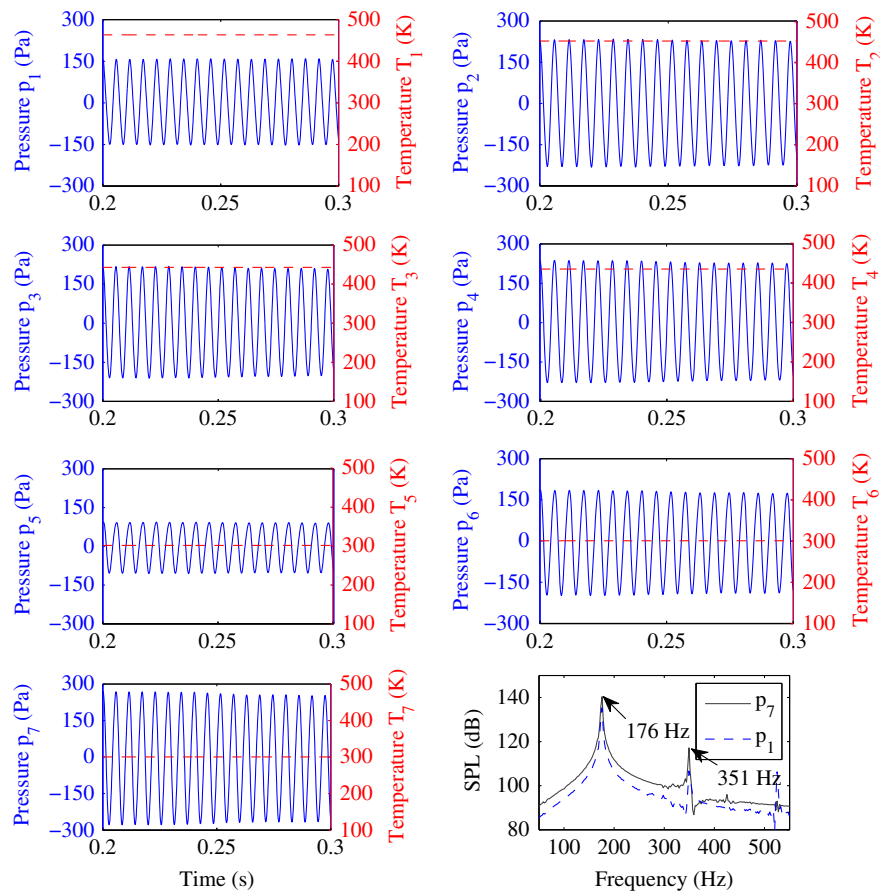


Fig. 7. Measured thermoacoustic oscillations and temperatures along the bifurcating branches of the Rijke–Zhao tube.

where $c_m^2 = \Gamma_m / \rho_m$ and $\kappa^2 = \frac{\omega^2}{c_m^2} - \frac{k_m}{c_m^2 m_m} - \frac{j\omega \zeta_m}{c_m^2 m_m}$. The diaphragm damping is generally sufficiently weak and so $\zeta_m \approx 0$.

To solve Eq. (18), Green's function approach is applicable and we can solve the partial differential equation

$$\frac{1}{r} \frac{\partial g}{\partial r} + \frac{\partial^2 g}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 g}{\partial \theta^2} + \kappa^2 g = -\frac{\delta(r - \mathcal{R})\delta(\theta - \phi)}{r} \quad (19)$$

where $0 < r, \mathcal{R} < a$ and $0 \leq \theta, \phi \leq 2\pi$, with the boundary conditions

$$\lim_{r \rightarrow 0} |g(r, \theta | \mathcal{R}, \phi)| < \infty \quad g(a, \theta | \mathcal{R}, \phi) = 0 \quad 0 \leq \theta, \phi \leq 2\pi \quad (20)$$

Since $\delta(\theta - \phi)$ can be expressed in Fourier cosine series [41] as

$$\delta(\theta - \phi) = \frac{1}{2\pi} \sum_{m=0}^{\infty} \epsilon_m \cos[m(\theta - \phi)] \quad (21)$$

It is anticipated that

$$g(r, \theta | \mathcal{R}, \phi) = \sum_{m=0}^{\infty} \epsilon_m g_m(r | \mathcal{R}) \cos(m(\theta - \phi)) \quad (22)$$

where $\epsilon_{m=0} = 1$ and $\epsilon_{m \neq 0} = 2$.

Substituting Eq. (22) into Eq. (19) yields

$$\frac{1}{r} \frac{\partial g_m}{\partial r} + \frac{\partial^2 g_m}{\partial r^2} + \left(\kappa^2 - \frac{m^2}{r^2} \right) g_m = -\frac{\delta(r - \mathcal{R})}{2\pi r} \quad (23)$$

The right hand side of Eq. (23) can be expanded as a Fourier–Bessel series as

$$\frac{\delta(r - \mathcal{R})}{2\pi r} = \frac{1}{\pi a^2} \sum_{n=1}^{\infty} \frac{J_m(k_{mn} \mathcal{R}/a) J_m(k_{mn} r/a)}{J_m^2(k_{mn})} \quad (24)$$

where k_{mn} is the n th root of Bessel functions of order m of the first kind $J_m(k) = 0$. The prime denotes spatial derivative. Because the delta function is a Fourier–Bessel series, the solution to Eq. (23) is anticipated in the form of

$$g_m(r | \mathcal{R}) = \sum_{n=1}^{\infty} \mathcal{A}_{mn} J_m \left(\frac{k_{mn} r}{a} \right) \quad (25)$$

Substituting Eqs. (25) and (24) into Eq. (23) gives rise to

$$\mathcal{A}_{mn} = \frac{1}{\pi a^2} \frac{J_m(k_{mn} \mathcal{R}/a)}{(k_{mn}^2/a^2 - \kappa^2) J_m^2(k_{mn})} \quad (26)$$

Therefore, the green's function of the diaphragm motion is given as

$$g(r, \theta | \mathcal{R}, \phi) = \frac{1}{\pi a^2} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{J_m(k_{mn} \mathcal{R}/a)}{(k_{mn}^2/a^2 - \kappa^2) J_m^2(k_{mn})} \cos[m(\theta - \phi)] \quad (27)$$

Fig. 4 illustrates the Green function g (normalized with its maximum value) for a circular piezoelectric diaphragm fixed at the rim (i.e. $r = a$), as $\theta - \phi$ is varied from 0 to 2π . It can be seen that there are multiple resonant modes of the piezoelectric diaphragm. And (a) $m = 0, n = 1$ corresponds the fundamental mode with lowest frequency (≈ 290 Hz). If the frequency of the oncoming pressure waves is close to one of the piezoelectric diaphragm resonant modes, then the diaphragm motion is maximized and so the voltage/power output.

Fig. 5 shows the measured dynamic response of the piezoelectric diaphragm before and after tuning. It can be seen from Fig. 5b that the piezoelectric generator can increase the voltage output by 105% in comparison with that without tuning. This is due to the fact that the tuning results in the shift of the resonance frequency of the diaphragm from 290 Hz to around 180 Hz, as shown in Fig. 5a. The spectrum indicates that in order to maximize the power output from piezoelectric generator, the thermoacoustic oscillations must be at frequencies closed to the resonant frequencies of the piezo-diaphragm vibration.

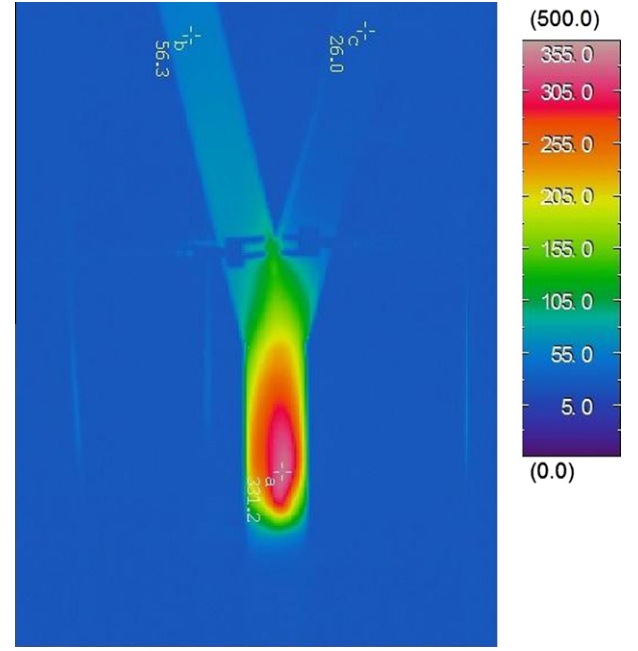


Fig. 8. Temperature measurement by using an infrared camera.

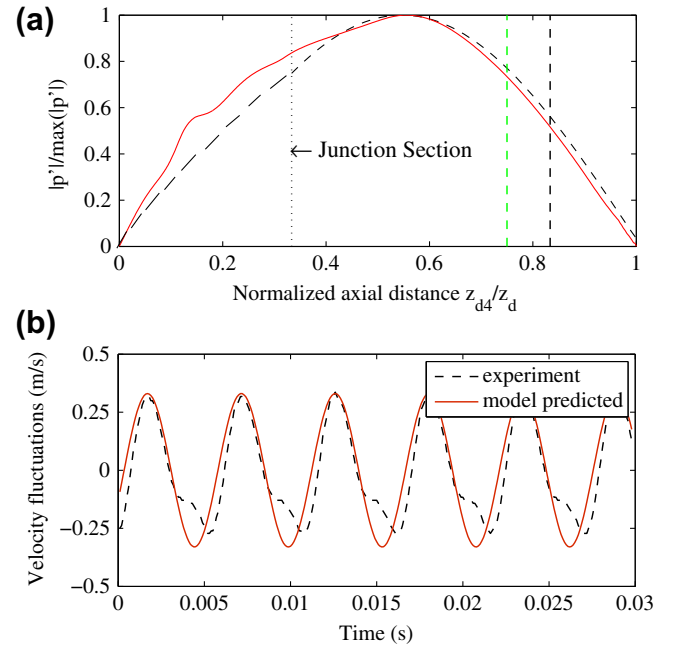


Fig. 9. (a) measured pressure mode shape along the upper longer branch and the mother tube and (b) Measurement and prediction of velocity oscillation near the piezo-diaphragm by using a hot wire anemometry.

3. Description of experiment

A Rijke–Zhao tube with a unique structure and geometry is designed and tested to demonstrate the thermoacoustic energy conversion. It has a mother tube (bottom stem), which splits into two daughter tubes (i.e. upper branches) with different lengths, see Fig. 6. As a heat source (flame, solar or electrical heater, etc.) is placed inside the mother tube, it provides a mechanism to produce convection-driven thermoacoustic oscillation. The geometric and dimensions of the Rijke–Zhao tube and piezo diaphragm is shown in the Table 1.

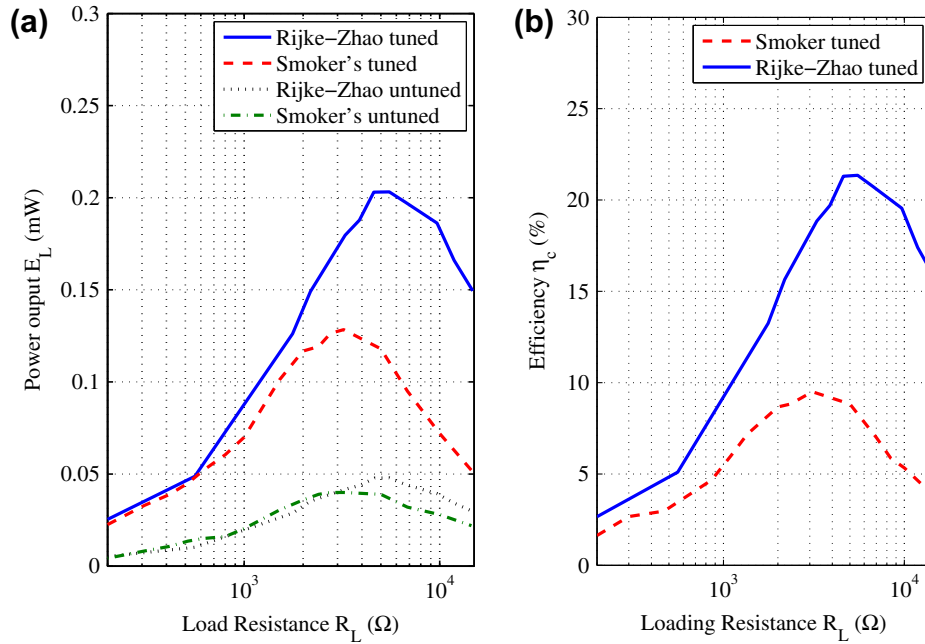


Fig. 10. Measured power output of the piezo-diaphragm and the acoustical energy conversion efficiency with varying resistance R_L in Ohms.

The experimental setup is shown in Fig. 6. T-shaped junctions¹ protruding from both bifurcating branches are designed for placing microphones and thermocouples. To measure pressure fluctuations along the Rijke-Zhao tube, two arrays of B&K microphones (Type 4794) are implemented: one array with 4 microphones applied to the longer upper branch, the other with 3 microphones attached to the shorter branch. The measurements are shown in Fig. 7. It can be seen that there are thermoacoustic oscillations in both branches, and the maximum pressure oscillation can reach about 139 dB. The dominant mode in Rijke-Zhao tube is at around 176 Hz and it has several harmonics as shown in the bottom graph. This frequency corresponds to the fundamental mode with a wavelength of twice of the total length of the bottom stem and the upper longer branch.

The temperature of the acoustic fields in the bifurcating upper branches are measured by using two arrays of K-type thermocouples placed side by side to the microphones. This is denoted by the dash line in Fig. 7. It can be seen that one branch is associated with self-sustained 'hot' oscillations (approximately 450 K), while the other one is with 'cold' oscillations at ambient temperature (≈ 300 K). This is due to the fact that 'hot' air goes through the upper longer branch, while the ambient air is 'sucked' into the other branch [39]. To further validate the temperature measurement, an infrared camera is used. It captures the quartz tube surface temperature spectrum. However, it can represent the 'hot' status of the sound waves inside each branch. The camera is set to operating range of 0–500 °C (corresponding to 273–773 K). Fig. 8 shows the measured temperature contour. It can be seen that the surface temperature of the short branch is around 26 °C (≈ 299 K), indicating that sound waves inside the branch are very close to room temperature. And this measurement is consistent with our thermocouples measurements. Note that the measurement error is $\pm 2^\circ$.

Finally, the 'oscillating' nature of thermoacoustics in the 'cold' branch is confirmed via direct measurement by using hot wire anemometry (HWA), as shown in Fig. 9b. The waveform deformation is most likely due to the presence of 2nd harmonic mode at about 360 Hz, as revealed by conducting spectrum analysis to the velocity measurement. The maximum volume flow rate is approximately $4.77 \times 10^{-4} \text{ m}^3/\text{s}$. To gain insights on the acoustic field, mode shape along the longer upper branch is estimated and shown in Fig. 9a. The total length of the upper branch and the mother tube equals half of the wavelength of the thermoacoustic oscillation. In addition, good agreement between numerical prediction and the measured one is observed. This confirms that Rijke-Zhao tube is a reliable device to produce self-sustained thermoacoustic oscillations to be utilized for energy harvesting.

4. Experimental measurement of output power and efficiency

The thermoacoustic oscillations at ambient temperature in the bifurcating branch enable the application of a piezoelectric generator to harvest thermal energy. In our experiment, a piezoelectric diaphragm (Piezo Systems Inc. T216-A4NO-573X) made of Lead-Zirconate-Titanate is used. It has diameter of 63.5 mm, thickness 0.41 mm and capacitance of 107 nF at resonant frequency of 290 Hz. Before attaching the piezoelectric diaphragm to the end of the upper short branch as show in Fig. 6, it is tuned to maximize its electricity output by adding an aluminum disk-plate weighing 2.85 g at the center of the diaphragm.

The performance of the convection-driven thermo-acoustic-piezo system is shown in Fig. 10. It can be seen from Fig. 10a that the maximum output power is about 2.1 mW. Compared with 1.28 mW from the conduction-driven thermo-acoustic-piezo system proposed by Smoker et al. [23], our present system produced 60% more power. In addition, tuning the piezo-diaphragm can greatly enhance the energy harvesting performance. Note that the electrical power output E_L is calculated by using

$$E_L = V_{\text{rms}}^2 / R_L = V^2 / 2R_L \quad (28)$$

¹ Note that the presence of T-shaped tubes might attenuate the thermoacoustic oscillations present due to boundary losses. However, they are not affecting the power output measurement from the piezoelectric membrane, since it is actuated by the pressure fluctuation at the end of the tube in the presence of T-shaped tubes. In practice, power output from the piezoelectric transducer might be increased by removing the T-shaped tubes.

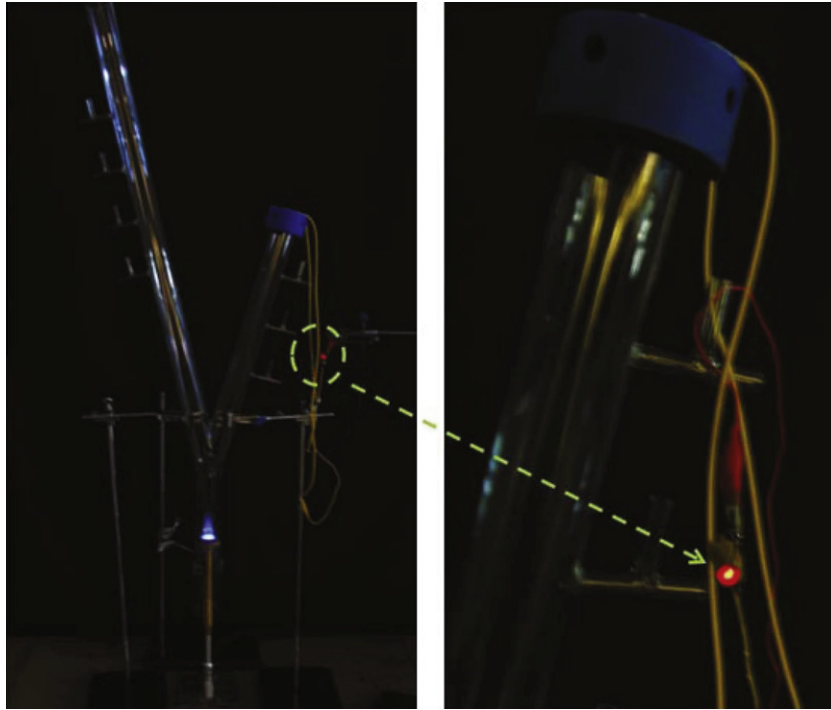


Fig. 11. A red LED is powered by the Rijke–Zhao thermo-acoustic-piezo system, which involves no heat exchangers and stacks (or regenerators). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where $V_{\text{rms}} = V/\sqrt{2}$ denotes the root mean square of the output voltage V . The acoustical energy conversion efficiency, η_c , is shown in Fig. 10b. The efficiency is defined as $\eta_c = E_L/p \cdot \mathcal{V}$, where p and $\mathcal{V}$ denote the pressure and volume velocity at the piezo-diaphragm position. It can be seen that the tuned piezoelectric diaphragm gives rise to the maximum energy conversion efficiency of approximately 22% when the load resistance is 5070 Ω . Compared with the maximum efficiency 9.8% obtained by Smoker et al. [23], our thermoacoustic system works 105% more efficient than the conduction-driven thermo-acoustic-piezo system. In addition, the overall efficiency of converting input heat power into output electrical power of conduction-driven system as designed by Smoker et al. [23] was shown to be about 0.00028%. In comparison with our present system, it is approximately 65% lower. The improvement of the overall efficiency is most likely due to the larger pressure oscillations used to excite the piezoelectric generator and increased acoustical energy conversion efficiency.

A practical demonstration is then made by using the electricity generated from the piezoelectric diaphragm to power a LED, as shown in Fig. 11. The successful application indicates that the Rijke–Zhao tube cannot only generate dual temperature thermoacoustic oscillations but also provide a platform for energy harvesting. It is worth noting that our present energy conversion system involves no heat exchangers and stack (or regenerators).

5. Discussion and conclusions

In summary, a nonlinear thermo-acoustic-piezo model is firstly developed to gain insight on the energy conversion process in our Rijke–Zhao system. It captures two physical processes: the generation and propagation of sound waves by inputting thermal energy

and the response of the piezoelectric diaphragm to sound waves. Then, a convection-driven thermo-acoustic-piezo system for energy harvesting is designed and experimentally tested. Unlike conventional conduction-driven thermoacoustic system, our present system is a convection-driven one. Furthermore, it does not involve any heat exchanger and stack (or regenerators). Comparing our present system performance with a similar but conduction-driven thermo-acoustic-piezo system reveals that the power output is increased by 60%. And acoustical energy conversion efficiency is increased by 105%. The experimental results confirm that our system is much simpler in design, lower cost in fabrication and energy-efficient.

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Appendix A. Matrices $\mathbf{U}$ and $\mathbf{V}$

The coefficient matrices $\mathbf{U}$ and $\mathbf{V}$ are given as

$$\mathbf{U} = \begin{pmatrix} 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & \frac{A_2(-1+\bar{M}_2)}{c_2} & -\frac{A_3}{c_3} & -\frac{A_4(1+\bar{M}_4)}{c_4} \\ 1+\bar{M}_1 \frac{\rho_1 c_1}{\rho_2 c_2} & -1+\bar{M}_1 \left(2-\frac{u_2}{u_1}-c_D\right)-\bar{M}_2^2 \left(1-\frac{u_2}{u_1}-\frac{c_D}{2}\right) & 0 & 0 & 0 \\ \frac{c_2}{c_1} \left(\frac{1+\gamma \bar{M}_2}{\gamma-1}\right) + \bar{M}_1 \bar{M}_2 \frac{\rho_1}{\rho_2} & \frac{1-\gamma \bar{M}_1}{\gamma-1} + \bar{M}_1^2 - \frac{\bar{M}_1^2}{2} (1-\bar{M}_1) \left(\frac{u_2^2}{u_1^2} - 1\right) & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbf{V} = \begin{pmatrix} -1 & 0 & 0 & -Z_3 & 0 \\ 0 & 0 & 0 & Z_3 & -Z_4 \\ \frac{-A_2(1+\bar{M}_2)}{c_2} & 0 & 0 & \frac{A_3(1-\bar{M}_3)}{c_3} Z_3 & \frac{A_4(1-\bar{M}_4)}{c_4} Z_4 \\ 0 & -\left(1 + \bar{M}_1 \left(2 - \frac{\bar{u}_2}{\bar{u}_1} - \frac{c_p}{2}\right) + \bar{M}_1^2 \left(1 - \frac{\bar{u}_2}{\bar{u}_1} - \frac{c_p}{2}\right)\right) Z_u & -\left(1 - \bar{M}_1 \frac{\bar{\rho}_1 c_1}{\bar{\rho}_2 c_2}\right) & 0 & 0 \\ 0 & -\left(\left(\frac{1+\gamma\bar{M}_1}{\gamma-1}\right) + \bar{M}_1^2 - \frac{\bar{M}_1^2}{2} (1 + \bar{M}_1) \left(\frac{\bar{u}_2^2}{\bar{b}_1^2} - 1\right)\right) Z_u & -\left(\frac{-\bar{c}_2}{c_1} \left(\frac{1-\gamma\bar{M}_2}{\gamma-1} - \bar{M}_1 \bar{M}_2 \frac{\bar{\rho}_1}{\bar{\rho}_2}\right)\right) & 0 & 0 \end{pmatrix}$$

Appendix B. Mechanical–electrical coupling piezoelectric generator

To maximize electric energy output, the piezoelectric generator operates at the resonance frequency of the vibrational structure to produce the maximum deformation of the piezoelectric materials. Near the resonance frequency, the mechanical behavior of the piezoelectric structure is well described by a one degree of freedom (1DOF) system. For simplicity, the single modal vibration of the mechanical structure is used to investigate the performance of the system. For a 1DOF system generator, such as a piezoelectric diaphragm vibrating under single bending modal, the mechanical–electrical coupling equations can be expressed as follows

$$m \frac{d^2 z}{dt^2} + c \frac{dz}{dt} - k_{me} v = \Delta p A_p \quad (29a)$$

$$q - k_{me} z - \frac{1}{c_p} v = 0 \quad (29b)$$

where m , c , k are modal mechanical mass, damping coefficient and stiffness respectively. k_{me} is the modal piezoelectric coupling stiffness, Δp is the modal external mechanical pressure applied, z is the modal displacement, v is the difference in electric potential between the electrodes, q is the electric charge on the electrodes, and c_p is the material capacitance. Taking Laplace transform to Eqs. (29a) and (29b) and manipulating them leads to the transfer function of $\hat{v}(s)/\Delta \hat{p}(s)$ as

$$\frac{\hat{v}(s)}{\Delta \hat{p}(s)} = \frac{A_p}{s^2 \left(\frac{C(s)}{k_{me}} - \frac{1}{c_p k_{me}} \right) m + s \left(\frac{C(s)}{k_{me}} - \frac{1}{c_p k_{me}} \right) c - k_{me}} \quad (30)$$

where $C(s) = \hat{q}(s)/\hat{v}(s)$. It can be seen that due to the coupling between the mechanical and electrical components, the exciting force give rise to the displacement in the piezo, which induces the electrical charge on the electrodes. This explains how the electrical power is generated from piezoelectric diaphragm as we used in our experiment.

References

- [1] Swift GW. Thermoacoustics: a unifying perspective for some engines and refrigerators. New York: Acoustical Society of America, American Institute of Physics; 2002.
- [2] Swift GW. thermoacoustic energy conversion. In: Rossing T, editor. The springer handbook of acoustics. Berlin, Heidelberg: Springer; 2007 [chapter 7].
- [3] Feldman KT. Review of the literature on Rijke thermoacoustic phenomena. J Sound Vib 1968;7:83–9.
- [4] Raun RL, Beckstead MW, Finlison JC, Brooks KP. Prog Energy Combust Sci 1993;19(5):313–64.
- [5] Feldman KT. Review of the literature on Sondhauss thermoacoustic phenomena. J Sound Vib 1968;7(1):71–82.
- [6] Bisio G, Rubatto G, Sondhauss and Rijke oscillations—thermodynamic analysis, possible applications and analogies. Energy 1999;24(1):117–31.

- [7] Markides CN, Smith TC. Energy 2011;36(12):6967–80.
- [8] Rott N. Thermo-acoustics. Adv Appl Mech 1984;20(1):135–75.
- [9] Moldenhauer S, Thess A, Holtmann C, Fernandez-Aballi C. Thermodynamic analysis of a pulse tube engine. Energy Convers Manage. <http://dx.doi.org/10.1016/j.enconman.2012.03.013>.
- [10] Rayleigh JWS. The theory of sound, vol. II. New York: Dover; 1945.
- [11] Culick FEC. A note on Rayleigh's criterion. Combust Sci Technol 1987;56(4):159–166.
- [12] Chu BT, Ying SJ. Thermally driven nonlinear oscillations in a pipe with travelling shock waves. Phys Fluids 1963;6(11):1625–37.
- [13] Zhao D, Morgans AS. Tuned passive control of combustion instabilities using multiple Helmholtz resonators. J Sound Vib 2009;320(2):744–57.
- [14] Zhao D. Transient growth of flow disturbances in triggering a Rijke tube combustion instability. Combust Flame 2012;159:2126–37.
- [15] Zhao D, Li J. Feedback control of combustion instabilities using a Helmholtz resonator with an oscillating volume. Combust Sci Technol 2012;184:694–716.
- [16] Zhong Z, Zhao D. Time-domain characterization of the acoustic damping of a perforated liner with bias flow. J Acoust Soc Am 2012;132:271–81.
- [17] McManus KR, Poinot T, Candel SM. A review of active control of combustion instabilities. Prog Energy Combust 1993;19:1–29.
- [18] Dowling AP, Morgans AS. Feedback control of combustion oscillations. Annu Rev Fluid Mech 2005;37:151–82.
- [19] Swift G. Thermoacoustic engines. J Acoust Soc Am 1988;84(4):1145–80.
- [20] Ceperley PH. A pistonless stirling engine – the traveling wave heat engine. J Acoust Soc Am 1979;66(1):1508–13.
- [21] Qiu L, Chen G, Jiang N. Optimum packing factor of the stack in a standing-wave thermoacoustic prime mover. Int J Energy Res 2002;26:729–35.
- [22] Sun D, Xu Y, Chen H, Wu K, Liu K, Yu Y. A mean flow acoustic engine capable of wind energy harvesting. Energy Convers Manage <http://dx.doi.org/10.1016/j.enconman.2011.12.035>.
- [23] Smoker J, Nough M, Aldraihem O, Baz A. Energy harvesting from a standing wave thermoacoustic-piezoelectric resonator. J Appl Phys 2012;111:104901.
- [24] Yu ZB, Jaworski AJ. Impact of acoustic impedance and flow resistance on the power output capacity of the regenerators in travelling thermoacoustic engines. Energy Convers Manage 2010;51(2):350–9.
- [25] Qiu L, Sun D, Tan Y, Yan W, Chen P, Zhao L, et al. Effect of pressure disturbance on onset process in thermoacoustic engine. Energy Convers Manage 2006;47:1383–90.
- [26] Yazaki T, Iwata A, Maekawa T, Tominaga A. Travelling wave thermoacoustic engine in a looped tube. Phys Rev Lett 1998;81(1):3128–31.
- [27] Hu ZJ, Li ZY, Li Q, Li Q. Evaluation of thermal efficiency and energy conversion of thermoacoustic stirling engines. Energy Convers Manage 2010;51(4):802–12.
- [28] Sun D, Qiu L, Zhang W, Yan W, Chen G. Investigation on travelling wave thermoacoustic heat engines with high pressure amplitude. Energy Convers Manage 2005;46(2):281–91.
- [29] Hu ZJ, Li Q, Li ZY, Li Q. Orthogonal experimental study on high frequency cascade thermoacoustic engines. Energy Convers Manage 2008;49(5):1211–7.
- [30] Zhou G, Li Q, Li ZY, Li Q. A miniature thermoacoustic stirling engine. Energy Convers Manage 2008;49(6):1785–92.
- [31] Rodriguez IA, Symko OG. Miniature traveling wave thermoacoustic engine. J Acoust Soc Am 2009;125(4):2562.
- [32] Babaei H, Siddiqui K. Design and optimization of thermoacoustic devices. Energy Convers Manage 2008;49(12):3585–92.
- [33] Hatazawa M, Sugita H, Ogawa T, Seo Y. Performance of a thermoacoustic sound wave generator driven with waste heat of automobile gasoline engine. Trans Jpn Soc Mech Eng 2004;16(1):292–9.
- [34] Shen C, He Y, Li Y, Ke H, Zhang D, Liu Y. Performance of solar powered thermoacoustic engine at different tilted angles. Appl Thermal Eng 2009;29(13):2745–56.
- [35] Chen RL, Garrett SL. The solar/heat driven thermoacoustic engine. J Acoust Soc Am 1998;103(5):2841–2.
- [36] Gardner DL, Swift GW. A cascade thermoacoustic engine. J Acoust Soc Am 2003;114(4):1905–19.

- [37] Zhao D, Barrow CA, Morgans AS, Carrotte J. Acoustic damping of a Helmholtz resonator with an oscillating volume. *AIAA J* 2009;47(7):1672–9.
- [38] Zhao D, Morgans AS, Dowling AP. Tuned Passive Control of the Acoustic Damping of Perforated Liners. *AIAA J* 2011;49(4):725–34.
- [39] Zhao D, Chow ZH. Dual-temperature combustion oscillations in Rijke–Zhao tube with two bifurcating branches. *J Fluid Mech*, submitted for publication.
- [40] Hagood NW, Flotow AV. Damping of structural vibrations with piezoelectric materials and passive electrical networks. *J Sound Vib* 1991;146(2): 243–68.
- [41] Zhao D. Transmission loss analysis of a parallel-coupled Helmholtz resonator network. *AIAA J* 2012;50(6):1339–46.